RELAXATION AUTO-OSCILLATIONS IN A BOREHOLE WITH A SAND PLUG

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Depending on borehole operation, a sand plug in the stem can exist in various states. The resistance to the fluid stream offered by a borehole whose plug is fluidized is a markedly different function of the discharge rate than the resistance of a borehole with a densely packed plug. Transition from the state in which the plug constitutes a dense layer of sand particles to a state with a fluidized plug occurs in a certain range of discharge rate values. The resistance to motion of the fluid in the borehole stem under a given set of conditions is a function which decreases with increasing discharge rate over a certain range of values of the latter. This results in instability of steady borehole gushing states associated for discharge rate values in this range and gives rise to auto-oscillatory gushing.

In the present paper we construct a model of the steady characteristic of a borehole with a sand plug and carry out an elementary analysis of the conditions of violation of steady-state stability and of the models of the resulting auto-oscillatory cycles.

§1. Let us consider a borehole of radius r_0 and depth H containing a sand plug. We assume that the volume concentration of sand in the densely packed plug is $\rho = \rho_*$ (the quantity ρ_* is usually close to 0.6), and that the height of the densely packed plug is h_0 (which usually ranges from several tens to several hundreds of meters). We also assume that the pressures at the mouth of the borehole and at its contour within the layer are constant and equal to p^0 and p_k, respectively. The pressure $p_0(t)$ in the borehole depends on time; it is clear that $p_k > p_0(t) > p^0$. Let us consider the resistance offered by the borehole stem to the fluid stream under the two sets of operating conditions (i.e., with a densely packed and a fluidized plug). We assume that the motion of the fluid is steady and that the liberation of dissolved gas at a certain depth in the stem is negligible.

1. Let the plug be densely packed. In this case the total resistance to motion consists of the ordinary hydraulic resistance to a fluid stream in a pipe of radius r_0 and length $H - h_0$ and the filtration resistance of the fluid in the plug. Introducing the corresponding fluid discharge functions per unit pipe length $\psi(q)$ and $\varphi(q)$ normalized to a unit area of the transverse cross section of the pipe, we obtain

$$p_0 - p^0 = \psi(q)(H - h_0) + \varphi(q)h_0 + d_1 g(H - h_0\rho_*). \quad (1.1)$$

Here d_1 is the density of the fluid, g is the free-fall acceleration, and the last term is the weight of the fluid column per unit cross-sectional area of the borehole. The functions $\varphi(q)$ and $\psi(q)$ can both be regarded as power functions of q. With laminar filtration in the plug and turbulent flow in the free portion of the borehole we have $\varphi(q) \sim q$, $\psi(q) \sim q^2$.

2. If the sand plug is fluidized, the total resistance can be expressed as a sum of the aforementioned hydraulic pressure and the resistance of the suspended layer consisting of plug particles. The latter can be assumed equal to the total weight of the suspended particles constituting the plug. This means that instead of (1.1) we have

$$p_0 - p^0 = \psi(q)(H - h(q)) + d_1 g(H - h_0\rho_*) + d_2 g h_0 \rho_*. \quad (1.2)$$

Here d_2 is the density of the particle material. For moderately large q we can assume [1] that $h(q) - h_0 \ll h_0$, so that $h(q) \approx h_0$. It is convenient to choose a pressure scale which ensures identical fulfillment of the equation

$$p^0 = -d_1 g(H - h_0\rho_*).$$

Equations (1.1) and (1.2) then, respectively, yield the relations

$$p_0 = F_1(q) = \psi(q)(H - h_0) + \varphi(q)h_0,$$

$$p_0 = F_2(q) = \psi(q)(H - h_0) + d_2 g h_0 \rho_*. \quad (1.3)$$

It is clear that relations (1.3) can also be used to describe quasi-steady-state motions such that $p_0(t)$ and $q(t)$ vary quite slowly. The volume concentration of sand in the fluidized plug can be determined from the relation

$$f(q, \rho) = mg. \quad (1.4)$$

Here m is the mass of a single particle and f is the viscous friction force exerted on each particle by the fluid stream. Many empirical expressions have been proposed for $f(q, \rho)$ [1].

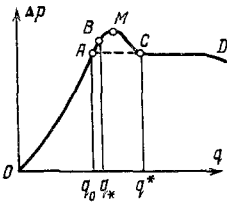

Fig. 1

§2. Let us consider qualitatively the process accompanying transition of the plug from the densely packed to the fluidized state. To this end we turn to the familiar dependence of the pressure drop in the suspended layer on the liquid-phase discharge rate [1, 2]. As q increases from zero, the difference Δp in the dense particle layer increases monotonically (Fig. 1). The layer begins to enter the fluidized state once q reaches and continues to increase beyond a certain critical value of q_*; at the same time the difference Δp decreases to a value approximately equal to the weight of all the particles in the layer. This state of transition from a dense to a suspended layer corresponds to the segment BC of the curve in Fig. 1. The difference Δp does not depend on q for $q > q^*$, and begins to decrease again only for very large q. This is usually attributed to the removal of individual particles from the layer (region D of the curve in Fig. 1). With reverse decrease in q, the representing point moves to the left along the curve CD. On reaching point C where $q = q^*$ it does not leave the segment BC of the fluidization characteristic, but moves along an extension of the line CD to its intersection with the curve OB at the point A, where $q = q_0$. With further decreases in q the value of Δp drops to zero along the same curve OB. The total change in the pressure difference in passing from the state $M(\Delta p_m = \max\{\Delta p\})$ to the state C for a suspended particle layer in a homogeneous fluid usually constitutes several percent of the quantity Δp at the point C; the delay in fluidization described by the segment ABMC on the curve in Fig. 1 is usually attributed to the presence of cohesive forces both between individual particles and between the particles and the walls of the apparatus in which fluidization is produced [1, 2]. If these forces were not present, transition to the fluidized state would occur abruptly at $q = q_0$.

We note that there have been attempts to attribute the additional pressure difference in the fluidization characteristic to acceleration of the particle materials with expansion of the dense layer. It appears that this factor can become significant only with very rapid ("explosive") expansion of the layer. Nevertheless, the additional difference has been noted in experiments with rather slow fluidization [1].

The presence of hysteresis in the fluidization characteristic is attributable to the fact that the characteristic time of this variation is small as compared with the relaxation time of the cohesive forces during the establishment of contacts between particles. Moreover, in experiments with particles coated with films of one fluid and fluidized

by a flow of another fluid (e.g., in the fluidization of wetted particles by air) the hysteresis can also be attributed to the fact that the intensification of transfer processes with transition to the suspended state is accompanied by partial thinning of the fluid films due to their evaporation or solution. We note that transition from a particle layer in a homogeneous fluid to a layer of wetted particles is associated with an increase in the value of the additional pressure difference and with an extension of the transient portion ABMC of the curve.

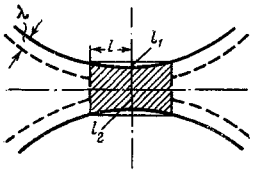

Fig. 2

With films of finite thickness covering the particles, homogeneous broadening of the layer gives rise to a gradual decrease in the cross-sectional area of the liquid meniscuses in the particle contact zone. Furthermore, loosening of the dense layer for $q \geqslant q_*$ makes possible the formation of local airholes and channels through which the excess fluidized medium can move. These factors have the effect of rupture of the adhesive bonds occurring at some rather than at all of the contacts between particles on passage through the critical value $q = q_*$ of the discharge rate. The fraction of ruptured bonds increases and monotonically approaches unity with increasing q, actually attaining it at approximately $q \approx q^*$. Thus, the intensity of the adhesive bonds does not drop abruptly to zero at $q = q_*$, but approaches zero gradually over some interval of discharge rate values $q_* < q \leqslant q^*$. This explains the widening of the declining portion of the fluidization characteristic. In this interval the remaining cohesive forces hinder further extension of the layer and its transition to the fluidized state. Equilibrium is established between the viscous friction forces appearing in (1.4) and the cohesive forces. The cohesive forces cease to play any significant role in the range $q > q^*$, and the relationship between q and the equilibrium value of the volume concentration ρ of the layer is given by Eq. (1.4).

In the transition range the total force exerted on all the suspended particles by the fluid stream turns out to be larger than the weight of all the particles constituting the layer. The chief role in this range is played by the downward forces of particle friction against the walls of the apparatus, which hinder the upward motion of the layer as a whole. Local "eruptions" of the fluid along random channels have the effect of increasing the friction forces, since they tend to increase the normal pressure against the walls of the apparatus. In general, if the friction forces are not sufficient to retain the layer, it is possible for the whole layer or a part of it to be ejected upward and for a piston-like state to become established. In this case it is possible to have an abrupt drop in Δp for $q \approx$ const by some amount δp, after which the Δp again varies smoothly. From now on we shall ignore the possibility of such a jump in the pressure difference, since its presence does not qualitatively alter our subsequent conclusions.

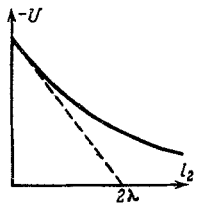

Fig. 3

As we know, petroleum filtered through a sand plug contains a certain proportion of heavy fractions (e.g., asphaltenes and other resins) characterized by considerable adhesive properties. These fractions are selectively adsorbed from the particle surfaces with the formation of sticky films differing in composition from the surrounding petroleum. These films can be regarded as analogs of the liquid films coating particles fluidized by another fluid. The cohesive forces between particles coated by sticky films increase sharply in comparison with the forces acting between the uncoated particles, and for sufficiently small particles may even exceed the weight of the latter. This can result in a further prolongation of the process of transition of the film from the densely packed to the fluidized state.

Sand particles from the plug are always present in the petroleum. Hence, apart from the possibility of mechanical deterioration of the sticky films on transition to the fluidized state, we can assume that their thickness remains constant. Recalling also the fact that the time required for the establishment of adhesive contact between the liquid film is quite small, we conclude that the variation of Δp with q is described by one and the same curve regardless of the direction of variation of q, i.e., the hysteresis noted above is virtually absent.

The analog of the quantity Δp in our problem is the pressure in the tapping face; the analogs of the segments OB and CD in the characteristic of Fig. 1 are the functions $F_1(q)$ and $F_2(q)$ in (1.3). To find the resistance function $F_3(q)$ in the transient range we must possess detailed knowledge of the composition of the petroleum, of the adhesive properties of its various fractions and their affinity for the particle material, and also of the microscopic particle shapes and dense packing structure. In addition, we must carry out a thorough investigation of fluctuation processes in the transient state. Because of this, it is hardly possible to derive $F_3(q)$ theoretically at the present time. We shall nevertheless quantitatively analyze the problem of determining $F_3(q)$ under several simplifying assumptions.

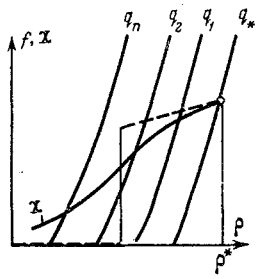

Fig. 4

§3. Let us consider the variation of the cohesive forces with homogeneous expansion of the dense layer. This expansion is accompanied by the disruption of direct contacts between hard particle surfaces and are replaced by contacts between the liquid films surrounding the particles. For simplicity we assume that the particles are spheres of radius a and that the minimum distance between their surface for a given expansion of the layer, when we can still speak of adhesive contact, constitutes $2l_2 \ll a$. We note that these assumptions already represent a major exaggeration of the true adhesive-contact area, since a) the quantity ρ_* for a layer of irregularly shaped particles is usually smaller than the ρ_* for a layer of ideal spherical particles [1], and b) the particle contact is approximated more closely by contact between a sharp prominence and a solid plane than by contact between two spheres.

The contact zone is characterized by a meniscus formed on merging of the liquid films whose thickness $\lambda \ll a$. It is convenient to replace the real meniscus by a figure whose cross section appears as the shaded area in Fig. 2 representing the contact zone. The volume of one-half of this figure under the assumption that $l \ll a$ is given by

$$\theta \approx \pi l^2 (l_1 + l_2) - \pi l_1^2 a \approx \pi l^2 l_2 + {}^1/_4 \pi l^4 a^{-1} .$$

On the other hand, this volume must equal the volume of the segment "cut out of" the liquid film by the meniscus, i.e.,

$$\theta \approx \pi\lambda (l_1^2 + 2l^2) \approx 2\pi\lambda l^2 .$$

Comparing these expressions, we obtain the following expressions for l and for the area s of a single contact:

$$l \approx 2[a\,(2\lambda - l_2)]^{1/2}, \quad s \approx 4\pi a\,(2\lambda - l_2), \quad l_2 \leqslant 2\lambda. \quad (3.1)$$

Replacement of the real meniscus by the above model also exaggerates the true minimum cross-sectional area s of the meniscus. The average area of a particle of grainy material is clearly given by

$$S \approx S_0 - zs + 2\pi l\,(l_1 + l_2 - \lambda)\,z \approx$$
$$\approx S_0 - 4\pi z a\,(2\lambda - l_2)\,Y\,(2\lambda - l_2),$$
$$S_0 = 4\pi\,(a + \lambda)^2 \approx 4\pi a^2. \quad (3.2)$$

Here z is the coordination number of the lattice approximating the given packing of the dense layer (the average number of contacts per particle); Y(x) is the Heaviside function.

The potential energy U per particle associated with the existence of adhesive forces in the medium is given by the expression

$$U = -(S_0 - S)\,\varkappa\sigma = -4\pi z a\,(2\lambda - l_2)\varkappa\sigma\,Y\,(2\lambda - l_2). \quad (3.3)$$

Here σ is the surface tension at the boundary between the sticky film and the ambient medium; ϰ is the coefficient which corrects for the adopted approximations. A rough estimate shows that ϰ is approximately one order smaller than unity. Moreover, if ϰ is also made to allow for the fact that the rupture of the adhesive bonds begins with the weakest contacts, and that the distribution of contacts over the average cross-sectional area of the meniscuses must in fact involve a large dispersion, it must be two, three, or even more orders smaller than unity.

The generalized force per particle associated with the change δl_2 in the distance between particles is (by (3.3)) given by

$$X' = 4\pi z a\varkappa\sigma Y\,(2\lambda - l_2). \quad (3.4)$$

Thus, in the approximation of homogeneous expansion of the particle layer the potential energy U of (3.3) depends linearly on l_2 (the broken line in Fig. 3), and the force X' of (3.4) is constant for $l_2 < 2\lambda$; both quantities vanish for $l_2 > 2\lambda$. In fact, as the concentration increases from the quantity ρ_*, the system begins to experience fluctuations in ρ whose intensity rises rapidly with increasing deviation. These fluctuations disrupt the homogeneity of expansion, lead to channel formation, etc. As a result, instead of one specific value of l_2 characterizing the layer at a given stage of expansion we have an entire spectrum of possible values of l_2 describing the distance between various specific particles. In particular, even if $l_2 \geqslant 2\lambda$, the system nevertheless involves adhesive contacts which make the effective force X' different from zero. In other words, the energy U increases with increasing average l_2 more slowly than according to law (3.3) (see the solid curve in Fig. 3), while the force X' decreases monotonically from its maximum value as given by expression (3.4) (since for $l_2 \approx 0$ there are no fluctuations and since the expansion can be considered homogeneous) to zero as $l_2 \to \infty$. This can be allowed for by replacing the Heaviside function in (3.3) and (3.4) by the monotonically increasing function $G(\rho)$ ($G(\rho_*) = 1$).

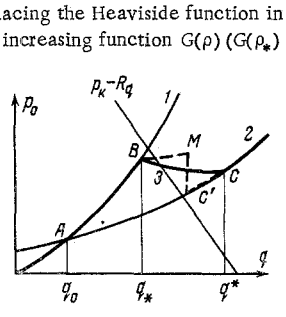

Fig. 5

Instead of l_2 we can take ρ or the specific particle volume ϑ as our generalized coordinate. Considering the functions $\rho(l_2)$ and $\vartheta(l_2)$, we can readily obtain expressions for the corresponding generalized forces,

$$X = X'\,(d\rho/dl_2)^{-1}, \qquad X'' = X'\,(d\vartheta/dl_2)^{-1}.$$

Multiplying X" by the numerical particle concentration n, we obtain an expression for the effective pressure of the system of packed particles considered in approximation of a continuous medium. It is this pressure occasioned by cohesive forces which hinders the "rupture" of such a continuous medium, i.e., passage of the particles into the suspended state. In this connection the external force acting on the particles can be considered roughly as a negative pressure.

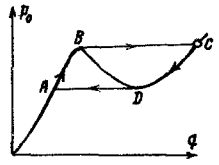

Fig. 6

Let the equilibrium state of a sand plug of concentration $\rho = \rho(q)$ be achieved for some discharge rate q. This state is clearly determined by the equality of the virtual work δA done by the external force to the change δU in the potential energy of the system. The work δA done by the force $f - mg$ over the particle displacements corresponding to the expansion of the plug (so that its concentration changes by δρ) turns out to be

$$\delta A \approx -\frac{2}{3}\frac{a^3}{\rho^2 r_0^2} N\,(f(q, \rho) - mg)\,\delta\rho.$$

Here N is the total number of sand particles in the plug. Recalling that $\rho \sim (a + l_2)^3$, we find from (3.3) that

$$\delta U \approx -{}^4/_3\pi\rho^{-1}a^2 z\varkappa\sigma N G\,(\rho)\delta\rho.$$

This yields an equation for determining $\rho = \rho(q)$,

$$f(q, \rho) = mg + 2\pi\rho a^{-1} z\varkappa\sigma G\,(\rho) = \chi\,(p). \quad (3.5)$$

Equation (3.5) coincides with Eq. (1.4) in the range of advanced fluidization ($G \approx 0$). The presence of cohesive forces between the particles is important during transition to the suspended state if

$$\tau = z\varkappa\sigma \gtrsim {}^2/_3 a^4 g\rho_*^{-1} d_2. \quad (3.6)$$

It is clear that this condition is satisfied over a broad range of values of a, σ, etc.

The solution of (3.5) is determined by the points of intersection of the curves $f(q, \rho)$ with the curve $\chi(\rho)$; the solution of (1.4) is determined by the points of intersection of the curves f with the axis of abscissas (see Fig. 4; the origin has been placed at the point $\chi = mg$, $q_n > \ldots > q_2 > q_1 > q_*$). The discharge rate q_0 introduced in Fig. 1 corresponds to the point $\rho = \rho_*$ on the axis of abscissas in Fig. 4. The function $\chi(\rho)$ for homogeneous expansion appears as the dashed line in Fig. 4.

We obtain the following expression for the required resistance function $F_3(q)$ of the borehole in the transient state:

$$p_0 = F_3\,(q) \approx \psi\,(q)\,(H - h_0) + n_*\chi\,(\rho) =$$
$$= \psi\,(q)\,(H - h_0) + d_2 g h_0 \rho_* + 2\pi n_* \rho a^{-1}\tau\,G\,(\rho). \quad (3.7)$$

Here n_* is the numerical particle concentration in the densely packed state. The appearance of a segment with a negative slope in the borehole characteristic F(q) is possible if the second and third terms in (3.7) are comparable in value and if condition (3.6) is fulfilled. The integral of the discharge rates over which F(q) is anomalous is clearly determined by the quantity λ and by the level of development of fluctuation processes in the system.

The steady-state characteristic of the borehole is shown in Fig. 5; curves 1, 2, and 3 correspond to the functions $F_1(q)$, $F_2(q)$, and $F_3(q)$ of (1.3) and (3.7). The slope of the curve to the right of point B is determined by the relationship of the various terms in (3.7) for $\rho = \rho(q_*) = \rho_*$. The approximation of homogeneous expansion of the layer corresponds to the dashed curve BMC' with the jump M → C' in Fig. 5.

§4. Let us assume that a filled layer is horizontal, that it is bounded by impermeable layers above and below, and that its thickness L coincides with the length of the tapping face of the borehole. The equation of transient filtration (in the elastic state) in the layer can be written as

$$\alpha \frac{\partial p}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(r \frac{\partial p}{\partial r}\right), \quad \alpha = \frac{m\mu c}{k}, \quad c = \frac{1}{d_1}\frac{dp}{d(d_1)}. \quad (4.1)$$

Here μ is the viscosity of the fluid, m is the porosity, k is the permeability of the layer, c is the compressibility of the fluid, and p(t,r) is the pressure in the layer. The boundary conditions are

$$p(t, r_k) = p_k, \quad p(t, r_0) = p_0(t). \quad (4.2)$$

The initial conditions are generally arbitrary. The volume discharge rate (the borehole discharge rate) is given by

$$q(t) = 2\pi r_0 L \frac{k}{\mu}\frac{\partial p(t, r)}{\partial r}\Big|_{r=r_0} = \frac{r_0}{\omega}\frac{\partial p(t, r)}{\partial r}\Big|_{r=r_0}. \quad (4.3)$$

In the steady state (4.1)–(4.3) yield the relations

$$p(t, r) = P(r), \quad q(t) = Q,$$
$$P(r_0) = p_k - RQ, \quad R = \omega \ln(r_k / r_0). \quad (4.4)$$

Here R is the constant resistance of the layer, and the quantities Q and $P_0 = P(r_0)$ are determined by the intersection of linear characteristic (4.4) of the layer with the borehole characteristic F(q) in Fig. 5. For small R and in the presence of a declining segment of the curve F(q) we can generally have three distinct steady borehole gushing states. For simplicity we assume from now on that only one steady state is realized.

The dynamic behavior of the quantities $p_0(t)$ and q(t) with deviations from the steady state is determined by the acceleration of the fluid and sand particles in the borehole stem, by the unsteady process of reconstruction of the pressure field in the layer, and by the finiteness of the rate of propagation of the perturbations. In accordance with what we said in §1 concerning the quantity h(q), the particle acceleration is negligible. We can also ignore the perturbation propagation time and the compressibility of the petroleum in the borehole stem. Applying Newton's second law, we derive an equation describing the dynamics of variation of the fluid discharge rate in the borehole stem,

$$\beta dq/dt = p_0(t) - F(q), \quad \beta \approx \pi r_0^2 (H - h_0\rho_*) d_1. \quad (4.5)$$

Let us investigate the stability of the steady states by the method developed in [3] for studying unsteady gushing states. Assuming small deviations of the tapping face pressure and discharge rate from their steady values, we find from (4.1)–(4.3), (4.5) that

$$\alpha \frac{\partial \Delta p}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(r \frac{\partial \Delta p}{\partial r}\right), \quad \Delta q = \frac{r_0}{\omega}\frac{\partial \Delta p}{\partial r}\Big|_{r=r_0},$$
$$\Delta p(t, r_k) = 0, \quad \Delta p(t, r_0) = \Delta p_0,$$
$$\beta \frac{d\Delta q}{dt} = \Delta p_0 - F'\Delta q, \quad F' = \frac{dF}{dq}\Big|_{q=Q}.$$

The corresponding characteristic equation is of the form

$$r_0(\beta\gamma + F')\sqrt{\alpha\gamma}\,[I_0(r_k\sqrt{\alpha\gamma})K_1(r_0\sqrt{\alpha\gamma}) +$$
$$+ I_1(r_0\sqrt{\alpha\gamma})K_0(r_k\sqrt{\alpha\gamma})] -$$
$$- \omega\,[I_0(r_0\sqrt{\alpha\gamma})K_0(r_k\sqrt{\alpha\gamma}) -$$
$$- I_0(r_k\sqrt{\alpha\gamma})K_0(r_0\sqrt{\alpha\gamma})] = 0. \quad (4.6)$$

Here γ is an ordinary characteristic variable ($\Delta q \sim \Delta p_0 \sim e^{\gamma t}$).

The method of [3] consists in introducing the new variable $u = r_0[\alpha\gamma]^{1/2}$, so that the stability condition becomes $|\arg u| > \pi/4$.

The roots γ of Eq. (4.6) are real for $\beta = 0$, since they constitute the eigenvalues of the self-adjoint Sturm-Liouville problem, and since the corresponding values of u lie on the imaginary axis. Entry into the domain of parameters of the problem where the stability conditions are violated is possible only at the points $\gamma = 0$ or $\gamma = \infty$ (by virtue of the realness of γ). Making use of expansions of the Bessel functions in (4.6) for small and large values of the argument, we obtain the following conditions for F':

$$F' > 0, \quad \text{or} \quad F' < -\omega \ln(r_k / r_0). \quad (4.7)$$

These conditions define the stability domain.

When the second condition of (4.7) is violated, stability loss can occur relative to arbitrarily small perturbations [3]. For $\beta \neq 0$, which corresponds to allowance for dynamic processes in the borehole stem, the second condition of (4.7) remains unaltered while the first is replaced by $F' > F_0' \neq 0$. The equation for determining F_0' is readily obtainable from (4.6) if we recall that at the point of entry into the stability domain we have $u = v + iv$, where v is real [3].

Thus, for certain F' the steady state of borehole operation becomes unstable and is replaced by an unsteady auto-oscillatory state. In view of the change in plug quality during the auto-oscillatory cycle, these auto-oscillations must be regarded as being of the relaxation type. The representing point in Fig. 5 alternately enters the domains $q < q_*$ and $q > q^*$ of the phase plane and remains there for periods of time of the same order as the characteristic time required for the state of the layer in the neighborhood of the tapping face to become altered. The latter time can be assumed to be considerably larger than the characteristic time of alteration of the flow state in the borehole [3]. This allows us to assume that the resistance of the borehole stem is quite accurately described by the steady characteristic F(q) in these ranges. On the other hand, the state of the sand plug changes in a time of the same order as the characteristic time of the borehole. Hence, from the standpoint of the processes occurring in the layer, the change in state of the plug can be assumed to occur jumpwise with constant pressure at the tapping face [3]. The approximate shape of the auto-oscillatory cycle is shown in Fig. 6. Segment AB of this cycle corresponds to operation of a borehole with a densely packed sand plug. In our case it replaces the segment of the ordinate axis in Fig. 6 which corresponds to the cessation of gushing and is associated with the auto-oscillations due to the liberation of dissolved gas at a certain depth in the borehole stem investigated in [3]. This fact generally enables one to ascertain the type of auto-oscillations associated with the gushing of a given borehole. It is clear that changes in layer conditions (e.g., lowering of the contour pressure) may lead to replacement of a steady state of borehole operation by an unsteady auto-oscillatory state, which in turn can be replaced by a steady state.

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